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VORTEX SOLUTIONS IN TWO-HIGGS-DOUBLET SYSTEMS

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Abstract

We analyze the existence of string-like defects in a two-Higgs-doublet system having $SU(2) \times U(1)_Y \times U(1)_{Y'}$ as gauge group. We are able to show that, when certain relations among the parameters hold, these configurations satisfy a set of first order differential equations (Bogomol'nyi equations) and their energy is proportional to their topological charge.

New interest in the study of string-like defects in spontaneously broken gauge theories has arisen after the observation made by Vachaspati [1] that the embedding [2] of the Nielsen-Olesen [3] string in the Standard Electroweak theory (model free of topological defects) is stable for a certain range of parameters. Nevertheless, the realistic values of the parameters, as derived from experiments, lie outside this range of classical stability [4].

This fact has led other authors to explore the existence of vortex solutions in extended versions of the standard model, containing a richer Higgs sector [5, 6, 7]. Dvali and Senjanovic have recently considered [5] a two-Higgs-doublet model with an additional $U(1)$ global symmetry which renders the vacuum manifold topologically non-trivial. Due to the global character of this extra symmetry topological configurations have (logarithmically) divergent energy.

In this letter, we will consider an $SU(2) \times U(1)_Y \times U(1)_{Y'}$ model, which corresponds to the model of [5], once the additional $U(1)$ symmetry is gauged.

We will be interested in the stability of finite energy strings in this model. The main result is that we are able to write a topological bound for the energy (Bogomol'nyi bound [8]) and, as a consequence, the stability of the configurations satisfying the bound is automatic. This bound is saturated when the fields satisfy a set of first order differential equations (self-dual or Bogomol'nyi equations, BE). As in the $U(1)$ case, the existence of BE severely constrains the form of the potential. This would be an unwanted feature of the model if it were not for the fact that in all models studied so far the existence of BE signals the presence of Supersymmetry (SUSY) [9]. In fact, our model has the same gauge group structure as that of SUSY extensions of the Weinberg-Salam Model that arise as low energy limits of E_6 based grand unified or superstring theories [10]. Nevertheless, the Higgs structure of these models is more complicated than in our case, due to the presence of additional $SU(2)$ singlets. Here, we shall content ourselves with our model which is the simplest extension of the Standard Model presenting BE and we leave for a future publication [11] the analysis of the connection with realistic SUSY extensions of the Standard Model.

The model we consider is described by the following Lagrangian density:

$$\mathcal{L} = \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} + \frac{1}{4}B_{\mu\nu} B^{\mu\nu} + \frac{1}{4}\tilde{B}_{\mu\nu} \tilde{B}^{\mu\nu} + \left|D_\mu^{(1)}\phi_1\right|^2 + \left|D_\mu^{(2)}\phi_2\right|^2 - V(\phi_1, \phi_2) \quad (1)$$

where $W_{\mu\nu}^a$, $B_{\mu\nu}$ and $\tilde{B}_{\mu\nu}$ are the field strengths associated with the gauge group $SU(2) \times U(1)_Y \times U(1)_{Y'}$

and the covariant derivatives are defined as ¹:

$$D_\mu^{(q)}\phi_q = \left(\partial_\mu + \frac{i}{2}g\tau^a W_\mu^a + \frac{i}{2}g'Y_q B_\mu + \frac{i}{2}g_1 Y'_q \tilde{B}_\mu \right) \phi_q. \quad (2)$$

In Eq.(2), τ_a are the Pauli matrices, the index $q = 1, 2$ labels the Higgs doublets ϕ_q and Y_q, Y'_q denote, respectively, the $U(1)_Y$ and $U(1)_{Y'}$ charges of ϕ_q .

The energy per unit length of axially symmetric static configurations is then given by:

$$E = \int d^2x \left[\frac{1}{4}W_{ij}^a W_{ij}^a + \frac{1}{4}B_{ij} B_{ij} + \frac{1}{4}\tilde{B}_{ij} \tilde{B}_{ij} + \left|D_i^{(1)}\phi_1\right|^2 + \left|D_i^{(2)}\phi_2\right|^2 + V(\phi_1, \phi_2) \right] \quad (3)$$

For the moment we will leave the potential $V(\phi_1, \phi_2)$ unspecified.

We start by using the standard Bogomol'nyi identity which consists in writing

$$\begin{aligned} \left|D_i^{(q)}\phi_q\right|^2 &= \frac{1}{2} \left|D_i^{(q)}\phi_q - i\gamma_q \epsilon_{ij} D_j^{(q)}\phi_q\right|^2 + \\ &+ \frac{1}{4}\gamma_q g \phi_q^\dagger \tau^a \phi_q \epsilon_{ij} W_{ij}^a + \frac{1}{4}\gamma_q g' Y_q \phi_q^\dagger \phi_q \epsilon_{ij} B_{ij} + \frac{1}{4}\gamma_q g_1 Y'_q \phi_q^\dagger \phi_q \epsilon_{ij} \tilde{B}_{ij} + \gamma_q \epsilon_{ij} \partial_i J_j^{(q)}. \end{aligned} \quad (4)$$

where $\gamma_q^2 = 1$ and the current is defined by:

$$J_j^{(q)} = \frac{1}{2i} \left[\phi^\dagger D_j^{(q)}\phi_q - (D_j^{(q)}\phi_q)^\dagger \phi_q \right]. \quad (5)$$

After using (4) and assuming that the current goes to zero at infinity, the energy can be rewritten as

$$\begin{aligned} E = E_{ST} + \int d^2x \frac{1}{4} &[W_{ij}^a + \epsilon_{ij} R^a]^2 + \frac{1}{4} [B_{ij} + \epsilon_{ij} R]^2 + \frac{1}{4} [\tilde{B}_{ij} + \epsilon_{ij} \tilde{R}]^2 + \\ &+ \frac{1}{2} \left|D_i^{(1)}\phi_1 - i\gamma_1 \epsilon_{ij} D_j^{(1)}\phi_1\right|^2 + \frac{1}{2} \left|D_i^{(2)}\phi_2 - i\gamma_2 \epsilon_{ij} D_j^{(2)}\phi_2\right|^2 + \\ &+ \left[V(\phi_1, \phi_2) - \frac{1}{2}R^a R^a - \frac{1}{2}R^2 - \frac{1}{2}\tilde{R}^2 \right] \end{aligned} \quad (6)$$

¹repeated space-time and gauge-group indices are summed over, while there is no summation on the index q, unless explicitly stated

where

$$R^a = \frac{g}{2}(\gamma_1 \phi_1^\dagger \tau^a \phi_1 + \gamma_2 \phi_2^\dagger \tau^a \phi_2) \quad , \quad (7)$$

$$R = \frac{g'}{2}(\gamma_1 Y_1 \phi_1^\dagger \phi_1 + \gamma_2 Y_2 \phi_2^\dagger \phi_2 - \rho) \quad , \quad (8)$$

$$\tilde{R} = \frac{g_1}{2}(\gamma_1 Y'_1 \phi_1^\dagger \phi_1 + \gamma_2 Y'_2 \phi_2^\dagger \phi_2 - \tilde{\rho}) \quad (9)$$

and

$$E_{ST} = \frac{1}{4} \int d^2x \epsilon_{ij} (g' \rho B_{ij} + g_1 \tilde{\rho} \tilde{B}_{ij}). \quad (10)$$

Here, ρ and $\tilde{\rho}$ are two arbitrary constants. We then see that if we choose the potential such that:

$$\begin{aligned} V(\phi_1, \phi_2) &= \frac{1}{2} R^a R^a + \frac{1}{2} R^2 + \frac{1}{2} \tilde{R}^2 = \\ &= \frac{1}{8} (g^2 + g'^2 Y_1^2 + g_1^2 Y_1'^2) (\phi_1^\dagger \phi_1)^2 + \frac{1}{8} (g^2 + g'^2 Y_2^2 + g_1^2 Y_2'^2) (\phi_2^\dagger \phi_2)^2 + \\ &+ \frac{1}{4} \gamma_1 \gamma_2 (g'^2 Y_1 Y_2 + g_1^2 Y'_1 Y'_2 - g^2) (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \frac{g^2}{2} \gamma_1 \gamma_2 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) + \\ &- \frac{1}{4} \gamma_1 (\rho g'^2 Y_1 + \tilde{\rho} g_1^2 Y'_1) (\phi_1^\dagger \phi_1) - \frac{1}{4} \gamma_2 (\rho g'^2 Y_2 + \tilde{\rho} g_1^2 Y'_2) (\phi_2^\dagger \phi_2) + \frac{\rho^2 g'^2}{8} + \frac{\tilde{\rho}^2 g_1^2}{8} \quad , \end{aligned} \quad (11)$$

the energy becomes a sum of squares plus a boundary term and then:

$$E \geq E_{ST} \quad . \quad (12)$$

The bound is reached if and only if the following Bogomol'nyi equations are satisfied:

$$W_{ij}^a = -\epsilon_{ij} R^a \quad , \quad (13)$$

$$B_{ij} = -\epsilon_{ij} R \quad , \quad (14)$$

$$\tilde{B}_{ij} = -\epsilon_{ij} \tilde{R} \quad , \quad (15)$$

$$D_i^{(1)} \phi_1 = i \gamma_1 \epsilon_{ij} D_j^{(1)} \phi_1 \quad , \quad (16)$$

$$D_i^{(2)} \phi_2 = i \gamma_2 \epsilon_{ij} D_j^{(2)} \phi_2 \quad . \quad (17)$$

In order to have finite energy configurations, the strengths of the gauge fields must vanish at infinity while the Higgs fields must satisfy the conditions

$$\lim_{r \rightarrow \infty} \phi_1(r, \theta) = v_1(\theta) \quad , \quad \lim_{r \rightarrow \infty} \phi_2(r, \theta) = v_2(\theta) \quad , \quad (18)$$

where

$$V[v_1(\theta), v_2(\theta)] = 0 \quad . \quad (19)$$

These conditions are met by requiring that:

$$R(v_1, v_2) = 0 \quad , \quad (20)$$

$$\tilde{R}(v_1, v_2) = 0 \quad , \quad (21)$$

$$R^a(v_1, v_2) = 0 \quad . \quad (22)$$

The first two equations fix ρ and $\tilde{\rho}$ in terms of $|v_1|^2$ and $|v_2|^2$:

$$\rho = (\gamma_1 Y_1 |v_1|^2 + \gamma_2 Y_2 |v_2|^2) \quad , \quad (23)$$

$$\tilde{\rho} = (\gamma_1 Y'_1 |v_1|^2 + \gamma_2 Y'_2 |v_2|^2) \quad . \quad (24)$$

On the other hand, Eq. (22) implies that

$$|v_1|^2 = |v_2|^2 \equiv \frac{v_0^2}{2} \quad (25)$$

and also determines the relative direction of $v_1(\theta)$ and $v_2(\theta)$.

In order to prove this, first notice that by means of a smooth gauge transformation one can always set one of the Higgs fields constant (at infinity):

$$v_1 = \frac{v_0}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad v_2 = \frac{v_0}{\sqrt{2}} \begin{pmatrix} A(\theta) \\ B(\theta) \end{pmatrix} \quad . \quad (26)$$

Eq. (22) then implies

$$A^* B = 0$$

$$1 = \gamma_1 \gamma_2 (|A|^2 - |B|^2) \quad . \quad (27)$$

There are two cases:

$$\text{case 1)} \quad \gamma_1 \gamma_2 = -1 \quad A = 0 \quad B = e^{i\chi(\theta)} \quad , \quad (28)$$

$$\text{case 2)} \quad \gamma_1 \gamma_2 = 1 \quad A = e^{i\chi(\theta)} \quad B = 0 \quad . \quad (29)$$

Since case 2) can be obtained from case 1) via an operation of charge conjugation on either one of the two Higgs fields, we will only consider, in the sequel of the paper, case 1).

We now see that the $SU(2)$ scalar $w(\theta) \equiv \frac{2}{v_0^2} \phi_1^\dagger \phi_2$ defines a map

$$w(\theta) = e^{i\chi(\theta)} : S_1 \rightarrow S_1 . \quad (30)$$

The winding n of this map is the topological charge of the configuration.

We will now show how the energy bound can be expressed in terms of this winding number. We saw earlier that at infinity the two Higgs fields have to be parallel. Via a smooth gauge transformation it is always possible to put them in the form:

$$\lim_{r \rightarrow \infty} \phi_q = \frac{v_0}{\sqrt{2}} \begin{pmatrix} 0 \\ e^{im_q \theta} \end{pmatrix} . \quad (31)$$

The topological charge is then equal to the relative winding of the Higgs fields:

$$n = m_2 - m_1 . \quad (32)$$

On the other hand since finite energy configurations are such that

$$\lim_{r \rightarrow \infty} D^{(q)} \phi_q = 0 \quad (33)$$

we obtain at infinity:

$$2m_1 - gW_\theta^3 + g'Y_1 B_\theta + g_1 Y_1' \tilde{B}_\theta = 0 , \quad (34)$$

$$2m_2 - gW_\theta^3 + g'Y_2 B_\theta + g_1 Y_2' \tilde{B}_\theta = 0 . \quad (35)$$

The energy bound

$$E_{ST} = \frac{1}{2} \pi v_0^2 \gamma_1 [g'(Y_1 - Y_2) B_\theta + g_1 (Y_1' - Y_2') \tilde{B}_\theta] \quad (36)$$

can then be expressed in terms of the topological charge as

$$E_{ST} = \pi v_0^2 \gamma_1 (m_1 - m_2) = -\pi v_0^2 \gamma_1 n . \quad (37)$$

Due to its topological character, n -vortex configurations saturating the bound are then necessarily stable.

Now, let us study in more detail such configurations. As we said, they satisfy the Bogomol'nyi Equations

(13-17). The simplest ansatz we can imagine is one where the Higgs are parallel in all space:

$$W_i^1 = W_i^2 = 0 \quad W_r^3 = B_r = \tilde{B}_r = 0 \quad ,$$

$$\phi_q = \frac{v_0}{\sqrt{2}} \begin{pmatrix} 0 \\ f_q(r) e^{im_q \theta} \end{pmatrix} \quad , \quad (38)$$

where

$$\lim_{r \rightarrow \infty} f_q = 1 \quad .$$

Eqs. (16-17) then become:

$$\partial_r f_q^2 = -\gamma_q \frac{1}{r} (2m_q - gW_\theta^3 + g'Y_q B_\theta + g_1 Y'_q \tilde{B}_\theta) f_q^2 \quad . \quad (39)$$

On the other hand, by taking linear combinations of Eqs. (13-15), we obtain

$$gg_1(Y_1 Y'_2 - Y_2 Y'_1) W_\theta^3 + gg_1(Y'_2 - Y'_1) B_\theta - g'g(Y_2 - Y_1) \tilde{B}_\theta = 0 \quad (40)$$

Eqs. (39-40) allow us to determine B_θ , \tilde{B}_θ and W_θ^3 once f_1 and f_2 are known. The equations for these last fields are found by inserting (39) in (13-15),

$$\nabla^2 \begin{pmatrix} \log f_1^2 \\ \log f_2^2 \end{pmatrix} = M \begin{pmatrix} f_1^2 - 1 \\ f_2^2 - 1 \end{pmatrix} \quad , \quad (41)$$

where M is the following mass matrix:

$$M = \frac{v_0^2}{4} \begin{pmatrix} g^2 + g'^2 Y_1^2 + g_1^2 Y_1'^2 & -g^2 - g'^2 Y_1 Y_2 - g_1^2 Y_1' Y_2' \\ -g^2 - g'^2 Y_1 Y_2 - g_1^2 Y_1' Y_2' & g^2 + g'^2 Y_2^2 + g_1^2 Y_2'^2 \end{pmatrix} \quad . \quad (42)$$

Eq. (41) is the generalization, for the two doublets model, of the $U(1)$ vortex equation [8]:

$$\nabla^2 \log f^2 = m^2(f^2 - 1) \quad . \quad (43)$$

Clearly, Eq. (41) only holds away of the zeroes of f_1 and f_2 . The behavior near the origin can be deduced from (39)

$$f_q = c_q r^{-2\gamma_q m_q}, \quad r \rightarrow 0 \quad (44)$$

and it then follows that, in order to have regular solutions, one must have

$$\gamma_q m_q \leq 0 \quad . \quad (45)$$

As we are working with $\gamma_1\gamma_2 = -1$, this implies

$$m_1 m_2 \leq 0 \quad .$$

As for the behavior at infinity, we can write

$$f_q(r) = 1 + h_q(r) \quad , \quad |h_q| \ll 1 \quad (46)$$

and then

$$\nabla^2 \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = M \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \quad . \quad (47)$$

This implies that:

$$f_q = 1 + c_q K_0(m_- r) \quad ,$$

where m_- is the lowest eigenvalue of the mass matrix.

Notice that within our ansatz there is a degeneracy associated with the different possible splitting of the topological charge between the two Higgs. In fact, for any n there are $|n| + 1$ different assignments of m_1 and m_2 compatible with Eq. (45). It is easy to verify that solutions with different (m_1, m_2) , although topologically equivalent are not related by a gauge transformation.

To conclude, let us analyze the mass spectrum of the theory. This can be easily done in the unitary gauge

$$\phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi \\ v_0 + h_1^0 \end{pmatrix} \quad \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} -\chi \\ v_0 + h_2^0 \end{pmatrix} \quad . \quad (48)$$

where χ is a complex field and h_1^0 and h_2^0 are real. With the potential given by (11), one finds:

$$m_\chi^2 = \frac{1}{2} g^2 v_0^2 \quad (49)$$

while the masses of the neutral components can be obtained by diagonalizing the mass matrix M . Regarding the gauge boson sector, one can check that besides the photon there is one charged particle, W^+ , and two neutral ones, Z_1 and Z_2 , whose masses are two by two equal to those of the corresponding Higgs particles. This phenomenon is the analogue of the one occurring in the $U(1)$ case where Bogomol'nyi equations exist only when the mass of the Higgs is equal to the mass of the gauge boson.

Summarizing, we have been able to show the existence of stable string-like solutions of arbitrary topological charge. The result follows from the possibility of writing a topological bound for the energy. We

have proposed a simple ansatz for the solutions of the BE, and even in this case they do not correspond to an embedding of the Nielsen-Olesen vortex. It would be interesting to know if there are more general solutions, exhibiting in particular the phenomenon of W -condensation [12]. Finally, the most interesting open problem is to understand the connection of our model with realistic SUSY extensions of the Standard Model. This issue is under current investigation [11].

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